

Induced Matter Brane Gravity and Einstein Static Universe

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Abstract

We investigate stability of the Einstein static universe against the scalar, vector and tensor perturbations in the context of induced matter brane gravity. It is shown that in the framework of this model, the Einstein static universe has a positive spatial curvature. In contrast to the classical general relativity, it is found that a stable Einstein static universe against the scalar perturbations does exist provided that the variation of time dependent geometrical equation of state parameter is proportional to the minus of the variation of the scale factor, $\delta\omega_g(t) = -C\delta a(t)$. We obtain neutral stability against the vector perturbations, and the stability against the tensor perturbations is guaranteed due to the positivity of the spatial curvature of the Einstein static universe in induced matter brane gravity.

1 Introduction

First attempts for finding a static solution of the field equations of general relativity to describe a homogenous and isotropic universe was done by Einstein which led him to the introduction of the cosmological constant [1]. Afterwards, some works have focused on the stability of the Einstein static universe against scalar, vector and tensor perturbations [2, 3, 4, 5, 6, 7]. As a whole, the Einstein static universe in the Einstein general relativity is unstable which means that it is almost impossible for the universe to maintain its stability during a long time because of the existence of varieties of perturbations such as the quantum fluctuations.

A renewed motivation for studying the Einstein static universe comes from the emergent universe scenario with the aim of solving the initial singularity problem of the standard model of cosmology [8]. In the framework of emergent universe model, the universe is originated from an Einstein static state rather than a big bang singularity. Observation from WMAP7 [9] supports the positivity of space curvature, in which it is found that a closed universe is favored at the 68% confidence level, and the universe stays past-eternally in an Einstein static state and then evolves to an inflationary phase. However, this cosmological model suffers from a fine-tuning problem which can be ameliorated by modifications to the cosmological equations of general relativity. For this reason, similar static solutions have been studied in the context of the modified theories of gravity such as $f(R)$ gravity [10, 11, 12], $f(T)$ gravity [13], Einstein-Cartan theory [14], nonconstant pressure models [15], Horava-Lifshitz gravity [16], Lyra geometry [17], loop quantum cosmology [18] and massive gravity [19]. Also, this model has been explored in braneworld scenarios inspired by string theory which describes gravity as a higher-dimensional theory so that it appears effectively 4-dimensional at low energy limits. In the framework of the braneworld scenarios, the standard gauge interactions are confined to the four-dimensional brane embedded in a higher dimensional bulk spacetime while the gravitational field propagates into the extra dimensions [20, 21, 22]. The Einstein static universe studied in the framework of braneworld scenarios such as [23, 24, 25, 26, 27]. As an instance, the author of [27] considered the induced Einstein field equations on the brane which is supported by an induced geometric energy-momentum and a confined matter field source on the brane. They obtained the stability conditions in terms of constant

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geometric linear equation of state parameter $\omega_{ext} = p_{extr}/\rho_{extr}$. Also, it is shown that for the case of radiation and matter dominated era, the stable Einstein static universe can be closed, open or flat in contrast to the vacuum energy dominated era in which the stable Einstein static universe can not be open.

In this work, we first have a brief review on induced matter brane gravity in which based on the Wesson induced matter theory [28], the higher dimensional spacetime (the bulk spacetime) is taken to have zero Ricci tensor. Then, we study stability of the Einstein static universe against the homogeneous scalar perturbations in the context of the induced matter brane gravity. Indeed, the induced field equations on the brane are studied with respect to the perturbation in the cosmic scale factor $a(t)$ and the geometric equation of state parameter $\omega_g(t)$. The evolution of the field equations are considered up to linear perturbations and all higher order terms are neglected. Also, the units of $8\pi G = 1$ are considered throughout this paper.

2 General Geometrical Setup of the Model

Consider the $4D$ background manifold \mathcal{M}_4 isometrically embedded in a n dimensional bulk \mathcal{M}_n by a differential map $\mathcal{Y}^A : \mathcal{M}_4 \rightarrow \mathcal{M}_n$ such that

$$\mathcal{G}_{AB}\mathcal{Y}_{,\mu}^A\mathcal{Y}_{,\nu}^B = \bar{g}_{\mu\nu}, \quad \mathcal{G}_{AB}\mathcal{Y}_{,\mu}^A\bar{\mathcal{N}}_a^B = 0, \quad \mathcal{G}_{AB}\bar{\mathcal{N}}_a^A\bar{\mathcal{N}}_b^B = g_{ab}, \quad (1)$$

where \mathcal{G}_{AB} ($\bar{g}_{\mu\nu}$) is the metric of the bulk (brane) space $\mathcal{M}_n(\mathcal{M}_4)$ in which $\{\mathcal{Y}^A\}$ ($\{x^\mu\}$) is the basis of the bulk (brane), $\bar{\mathcal{N}}_a^A$ are $(n-4)$ normal unit vectors orthogonal to the brane and $g_{ab} = \epsilon\delta_{ab}$ in which $\epsilon = \pm 1$ correspond to the two possible signature of each extra dimension. Perturbation of the background manifold \mathcal{M}_4 in a sufficiently small neighborhood of the brane along an arbitrary transverse direction ξ^a is given by

$$\mathcal{Z}^A(x^\mu, \xi^a) = \mathcal{Y}^A + (\mathcal{L}_\xi \mathcal{Y})^A, \quad (2)$$

where \mathcal{L}_{ξ^a} represents the Lie derivative along ξ^a where ξ^a with $a = 5, \dots, n$ are small parameters along \mathcal{N}^A_a parameterizing the non-compact extra dimensions. The presence of the tangent component of the vector ξ along the brane can cause some difficulties because it can induce some undesirable coordinate gauges. But, it is shown that in the theory of geometric perturbations, it is possible to choose this vector to be orthogonal to the background [29]. Then, by choosing the extra dimensions ξ^a to be orthogonal to the brane, we ensure gauge independency [30] and have perturbations of the embedding along the orthogonal extra directions \mathcal{N}^A_a , giving the local coordinates of the perturbed brane as

$$\begin{aligned} \mathcal{Z}_{,\mu}^A(x^\nu, \xi^a) &= \mathcal{Y}_{,\mu}^A(x^\nu) + \xi^a \mathcal{N}^A_{a,\mu}, \\ \mathcal{Z}_{,a}^A(x^\nu, \xi^a) &= \mathcal{N}^A_a. \end{aligned} \quad (3)$$

It is seen from equation (2) that since the vectors \mathcal{N}^A depend only on the local coordinates x^μ , $\mathcal{N}^A = \mathcal{N}^A(x^\mu)$, so they do not propagate along the extra dimensions

$$\mathcal{N}^A_a = \bar{\mathcal{N}}^A_a + \xi^b [\bar{\mathcal{N}}^A_a, \bar{\mathcal{N}}^A_b] = \bar{\mathcal{N}}^A_a. \quad (4)$$

The above assumptions lead to the embedding equations of the perturbed geometry as

$$\mathcal{G}_{AB}\mathcal{Z}_{,\mu}^A\mathcal{Z}_{,\nu}^B = g_{\mu\nu}, \quad \mathcal{G}_{AB}\mathcal{Z}_{,\mu}^A\mathcal{N}^B_a = g_{\mu a}, \quad \mathcal{G}_{AB}\mathcal{N}^A_a\mathcal{N}^B_b = g_{ab}. \quad (5)$$

where by setting $\mathcal{N}^A_a = \delta^A_a$, the metric of the bulk space \mathcal{G}_{AB} in the Gaussian frame and in the vicinity of \mathcal{M}_4 can be written in the following matrix form

$$\mathcal{G}_{AB} = \begin{pmatrix} g_{\mu\nu} + A_{\mu c}A^c_\nu & A_{\mu a} \\ A_{\nu b} & g_{ab} \end{pmatrix}. \quad (6)$$

Then, the line element of the bulk space is given by

$$dS^2 = \mathcal{G}_{AB}d\mathcal{Z}^Ad\mathcal{Z}^B = g_{\mu\nu}(x^\alpha, \xi^a)dx^\mu dx^\nu + g_{ab}d\xi^a d\xi^b, \quad (7)$$

where

$$g_{\mu\nu} = \bar{g}_{\mu\nu} - 2\xi^a \bar{K}_{\mu\nu a} + \xi^a \xi^b \bar{g}^{\alpha\beta} \bar{K}_{\mu\alpha} \bar{K}_{\nu\beta b}, \quad (8)$$

represents the metric of the perturbed brane, so that

$$\bar{K}_{\mu\nu a} = -\mathcal{G}_{AB} \mathcal{Y}^A_{,\mu} \mathcal{N}^B_{a;\nu} = -\frac{1}{2} \frac{\partial g_{\mu\nu}}{\partial \xi^a}, \quad (9)$$

is the extrinsic curvature of the original brane (the second fundamental form). We use the notation $A_{\mu c} = \xi^d A_{\mu cd}$, where

$$A_{\mu cd} = \mathcal{G}_{AB} \mathcal{N}^A_{d;\mu} \mathcal{N}^B_c = \bar{A}_{\mu cd}, \quad (10)$$

represents the twisting vector fields (the normal fundamental form). Any fixed ξ^a indicates a new perturbed brane and enables us to define an extrinsic curvature for this perturbed brane similar to the original one by

$$\tilde{K}_{\mu\nu a} = -\mathcal{G}_{AB} \mathcal{Z}^A_{,\mu} \mathcal{N}^B_{a;\nu} = \bar{K}_{\mu\nu a} - \xi^b (\bar{K}_{\mu\gamma a} \bar{K}^\gamma_{\nu b} + A_{\mu ca} A^c_{b\nu}). \quad (11)$$

Note that the definitions (6), (8) and (11) require

$$\tilde{K}_{\mu\nu a} = -\frac{1}{2} \frac{\partial \mathcal{G}_{\mu\nu}}{\partial \xi^a}. \quad (12)$$

In geometric language, the presence of gauge fields $A_{\mu a}$ tilts the embedded family of sub-manifolds with respect to the normal vector \mathcal{N}^A . According to our construction, the original brane is orthogonal to the normal vector \mathcal{N}^A . However, equation (5) shows that this is not true for the deformed geometry. Thus, we change the embedding coordinates as

$$\mathcal{X}^A_{,\mu} = \mathcal{Z}^A_{,\mu} - g^{ab} \mathcal{N}^A_a A_{b\mu}, \quad (13)$$

where the coordinates \mathcal{X}^A describe a new family of embedded manifolds whose members are always orthogonal to \mathcal{N}^A . In this coordinates the embedding equations of the perturbed brane is similar to the original one, described by equation (1), so that the coordinates \mathcal{Y}^A is replaced by \mathcal{X}^A . This new embedding of the local coordinates are suitable for obtaining induced Einstein field equations on the brane. The extrinsic curvature of a perturbed brane then becomes

$$K_{\mu\nu a} = -\mathcal{G}_{AB} \mathcal{X}^A_{,\mu} \mathcal{N}^B_{a;\nu} = \bar{K}_{\mu\nu a} - \xi^b \bar{K}_{\mu\gamma a} \bar{K}^\gamma_{\nu b} = -\frac{1}{2} \frac{\partial g_{\mu\nu}}{\partial \xi^a}, \quad (14)$$

which is the generalized York's relation and shows the propagation of the extrinsic curvature as a result of the metric propagation in the direction of extra dimensions. The components of the Riemann tensor of the bulk in the embedding vielbein $\{\mathcal{X}^A_{,\alpha}, \mathcal{N}^A_a\}$, lead to the Gauss-Codazzi equations [31]

$$R_{\alpha\beta\gamma\delta} = 2g^{ab} K_{\alpha[\gamma a} K_{\delta]\beta b} + \mathcal{R}_{ABCD} \mathcal{X}^A_{,\alpha} \mathcal{X}^B_{,\beta} \mathcal{X}^C_{,\gamma} \mathcal{X}^D_{,\delta}, \quad (15)$$

$$2K_{\alpha[\gamma c; \delta]} = 2g^{ab} A_{[\gamma a c} K_{\delta] \alpha b} + \mathcal{R}_{ABCD} \mathcal{X}^A_{,\alpha} \mathcal{N}^B_c \mathcal{X}^C_{,\gamma} \mathcal{X}^D_{,\delta}, \quad (16)$$

where \mathcal{R}_{ABCD} and $R_{\alpha\beta\gamma\delta}$ are the Riemann tensors of the bulk and the perturbed brane, respectively. One can find the Ricci tensor by contracting the Gauss equation (15) as

$$R_{\mu\nu} = (K_{\mu\alpha c} K_{\nu}{}^{\alpha c} - K_c K_{\mu\nu}{}^c) + \mathcal{R}_{AB} \mathcal{X}^A_{,\mu} \mathcal{X}^B_{,\nu} - g^{ab} \mathcal{R}_{ABCD} \mathcal{N}^A_a \mathcal{X}^B_{,\mu} \mathcal{X}^C_{,\nu} \mathcal{N}^D_b. \quad (17)$$

The next contraction will give the Ricci scalar as

$$R = \mathcal{R} + (K \circ K - K_a K^a) - 2g^{ab} \mathcal{R}_{AB} \mathcal{N}^A_a \mathcal{N}^B_b + g^{ad} g^{bc} \mathcal{R}_{ABCD} \mathcal{N}^A_a \mathcal{N}^B_b \mathcal{N}^C_c \mathcal{N}^D_d, \quad (18)$$

where we have denoted $K \circ K \equiv K_{a\mu\nu} K^{a\mu\nu}$ and $K_a \equiv g^{\mu\nu} K_{a\mu\nu}$. Using equations (17) and (18) we obtain the following relation between the Einstein tensors of the bulk and brane

$$G_{AB} \mathcal{X}^A_{,\mu} \mathcal{X}^B_{,\nu} = G_{\mu\nu} - Q_{\mu\nu} - g^{ab} \mathcal{R}_{AB} \mathcal{N}^A_a \mathcal{N}^B_b g_{\mu\nu} + g^{ab} \mathcal{R}_{ABCD} \mathcal{N}^A_a \mathcal{X}^B_{,\mu} \mathcal{X}^C_{,\nu} \mathcal{N}^D_b, \quad (19)$$

where G_{AB} and $G_{\mu\nu}$ are the Einstein tensors of the bulk and brane respectively, and

$$Q_{\mu\nu} = g^{ab}(K_{a\mu}{}^\gamma K_{\gamma\nu b} - K_a K_{\mu\nu b}) - \frac{1}{2}(K \circ K - K_a K^a)g_{\mu\nu}. \quad (20)$$

From the definition of $Q_{\mu\nu}$, it is an independent conserved geometrical quantity as $\nabla_\mu Q^{\mu\nu} = 0$ [32].

Using the decomposition of the Riemann tensor of the bulk space into the Weyl curvature tensor, the Ricci tensor and the scalar curvature as

$$\mathcal{R}_{ABCD} = C_{ABCD} - \frac{2}{n-2}(\mathcal{G}_{B[D}\mathcal{R}_{C]A} - \mathcal{G}_{A[D}\mathcal{R}_{C]B}) - \frac{2}{(n-1)(n-2)}\mathcal{R}(\mathcal{G}_{A[D}\mathcal{R}_{C]B}), \quad (21)$$

we obtain the induced 4D Einstein equation on the brane as

$$\begin{aligned} G_{\mu\nu} = & G_{AB}\mathcal{X}_{,\mu}^A\mathcal{X}_{,\nu}^B + Q_{\mu\nu} - \mathcal{E}_{\mu\nu} + \frac{n-3}{n-2}g^{ab}\mathcal{R}_{AB}\mathcal{N}_a^A\mathcal{N}_b^B g_{\mu\nu} \\ & - \frac{n-4}{n-2}\mathcal{R}_{AB}\mathcal{X}_{,\mu}^A\mathcal{X}_{,\nu}^B + \frac{n-4}{(n-1)(n-2)}\mathcal{R}g_{\mu\nu}, \end{aligned} \quad (22)$$

where $\mathcal{E}_{\mu\nu} = g^{ab}\mathcal{C}_{ABCD}\mathcal{X}_{,\mu}^A\mathcal{N}_a^B\mathcal{N}_b^C\mathcal{X}_{,\nu}^D$ is the electric part of the Weyl tensor of the bulk space \mathcal{C}_{ABCD} . The electric part of the Weyl tensor is well known from the brane point of view. It describes a traceless matter, denoted by dark radiation or Weyl matter.

3 Induced Matter Brane Gravity, the Einstein Static Universe and Stability Analysis

Now, let us to concentrate on the induced matter brane gravity. In this theory, the motivation for assuming the existence of large extra dimensions was to achieve the unification of matter and geometry, *i.e.*, to obtain the properties of matter as a consequence of extra dimensions. In the framework of the IMT, Einstein field equations in the bulk are written in the form of

$$\mathcal{R}_{AB} = 0, \quad (23)$$

where \mathcal{R}_{AB} is the Ricci tensor of the nD bulk space [28]. Then, using equations (22) and (23), the Einstein field equations induced on the brane become

$$G_{\mu\nu} = Q_{\mu\nu} - \mathcal{E}_{\mu\nu}. \quad (24)$$

Thus, from a 4D point of view, the empty nD field equations look like Einstein equations with induced matter source as

$$8\pi G_N T_{\mu\nu} = Q_{\mu\nu} - \mathcal{E}_{\mu\nu}. \quad (25)$$

In what follows, we restrict our analysis to a constant curvature bulk ($\mathcal{E}_{\mu\nu} = 0$) and will focus on the geometrical quantity, $Q_{\mu\nu}$, as the induced matter on the brane. As was mentioned before, $Q_{\mu\nu}$ is a conserved quantity and then can describe the ordinary matter fields with a geometrical origin in accordance with the spirit of the IMT. From this point of view, the induced matter fields on the 4D brane appears as the effects of a higher dimensional geometry.

For the purpose of embedding of the *FRW* brane in a five dimensional bulk space, one should consider the metric

$$ds^2 = -dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \quad (26)$$

where $a(t)$ is the cosmic scale factor, $k = +1, -1$ or 0 corresponds to the closed, open or flat universes and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the metric of 2-sphere. The components of the extrinsic curvature for the FRW metric are given by

$$\begin{aligned} K_{00} &= -\frac{1}{a}\frac{d}{dt}\left(\frac{b}{a}\right), \\ K_{ij} &= \frac{b}{a^2}g_{ij}, \quad i, j = 1, 2, 3. \end{aligned} \quad (27)$$

where dot means derivative with respect to the cosmic time t and $b = b(t)$ is an arbitrary function of it [32]. Then, by defining the parameters $h := \frac{\dot{b}}{b}$ and $H := \frac{\dot{a}}{a}$, the components of the induced geometric energy-momentum tensor on the brane, $Q_{\mu\nu}$, take the form of

$$\begin{aligned} Q_{00} &= \frac{3b^2}{a^4}, \\ Q_{ij} &= -\frac{b^2}{a^4} \left(\frac{2h}{H} - 1 \right) g_{ij}. \end{aligned} \quad (28)$$

Also, the geometric energy-momentum tensor $Q_{\mu\nu}$ can be identified as a perfect fluid as

$$Q_{\mu\nu} = (\rho_g + p_g)u_\mu u_\nu + p_g g_{\mu\nu}, \quad (29)$$

where $u_\alpha = \delta_\alpha^0$ and ρ_g and p_g denote the “geometric energy density” and “geometric pressure”, respectively (the suffix “ g ” stands for “geometric”). Then, using the equations (28) and (29) we obtain

$$\begin{aligned} \rho_g &= \frac{3b^2}{a^4}, \\ p_g &= -\frac{b^2}{a^4} \left(\frac{2h}{H} - 1 \right). \end{aligned} \quad (30)$$

In addition, the geometric fluid can be implemented by the equation of state $p_g = \omega_g \rho_g$ where ω_g is the geometric equation of state parameter and generally is a function of time, $\omega_g = \omega_g(t)$. Using equations (30) and the equation of state of the geometric fluid, we obtain the following differential equation

$$\frac{\dot{b}}{b} = \frac{1}{2} (1 - 3\omega_g) \frac{\dot{a}}{a}, \quad (31)$$

which yields the following solution for $b(t)$ in terms of the scale factor and the geometric equation of state parameter

$$b(t) = b_0 \left(\frac{a}{a_0} \right)^{\frac{1}{2}} \exp \left(-\frac{3}{2} \int \omega_g(t) \frac{\dot{a}}{a} dt \right), \quad (32)$$

where a_0 and b_0 are the scale factor and the curvature warp of the initial Einstein static universe, respectively.

Using equations (24) and (28), the first components of the induced Einstein equation on the brane will be

$$\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{b_0^2}{a_0 a^3} \exp \left(-3 \int \omega_g(t) \frac{\dot{a}}{a} dt \right), \quad (33)$$

in which for the Einstein static universe, described by the condition $\dot{a} = \ddot{a} = 0$, gives the spatial curvature as

$$k = \frac{b_0^2}{a_0^2}. \quad (34)$$

This equation denotes that for a Einstein static universe in the framework of induced matter brane gravity, the spatial curvature of spacetime must be positive.

Similarly, using the equations (24) and (28), the pressure component of the induced Einstein equation on the brane will be

$$-2 \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a} \right)^2 - \frac{k}{a^2} = \frac{3b_0^2}{a_0 a^3} \omega_g(t) \exp \left(-3 \int \omega_g(t) \frac{\dot{a}}{a} dt \right). \quad (35)$$

This equation, by using equation (34), gives us the equation of state parameter of the Einstein static universe as

$$\omega_{0g} = -\frac{1}{3}. \quad (36)$$

3.1 Scalar Perturbations

In what follows, we consider linear homogeneous scalar perturbations of equations (33) and (35) around the Einstein static universe described by the scale factor a_0 and explore its stability against these perturbations. The perturbation in the cosmic scale factor $a(t)$ and geometric equation of state parameter $\omega_g(t)$ can be considered as

$$\begin{aligned} a(t) &\rightarrow a_0(1 + \delta a(t)), \\ \omega_g(t) &\rightarrow \omega_{0g}(1 + \delta\omega_g(t)). \end{aligned} \quad (37)$$

Substituting these equations in equation (35) and linearizing the result gives the following equation

$$\delta\ddot{a} = \frac{k}{2a_0^2}\delta\omega_g, \quad (38)$$

where similar process on equation (33) does not yield any nontrivial result. Equation (38) represents that for having a stable Einstein static universe, the perturbation in the geometric equation of state parameter has to be in the form of $\delta\omega_g = -C\delta a$ where C is an arbitrary positive constant. In this case, the equation (38) has the solution

$$\delta a = C_1 e^{iAt} + C_2 e^{-iAt}, \quad (39)$$

where C_1 and C_2 are integration constants and $A = (\frac{kC}{2a_0^2})^{\frac{1}{2}}$ is the frequency of oscillation around the Einstein static universe.

It is seen that the results in the framework of induced matter brane gravity are different from the result obtained in a braneworld scenario with a confined matter field source on the brane [27]. In that work, it is shown that for the case of radiation and matter dominated era, the stable Einstein static universe can be closed, open or flat in contrast to the vacuum energy dominated era in which the stable Einstein static universe can not be open. But, in the framework of induced matter brane gravity, it is needed that the spatial curvature of the universe to be positive and the variation of the geometric equation of state parameter must be proportional to the minus of the variation of the scale factor.

3.2 Vector and Tensor Perturbations

In the cosmological context, the vector perturbations of a perfect fluid having energy density ρ and barotropic pressure $p = \omega\rho$ are governed by the co-moving dimensionless *vorticity* defined as $\varpi_a = a\varpi$. The vorticity modes satisfy the following propagation equation [33]

$$\dot{\varpi}_\kappa + (1 - 3c_s^2)H\varpi_\kappa = 0, \quad (40)$$

where $c_s^2 = dp/d\rho$ and H are the sound speed and the Hubble parameter, respectively. This equation is valid in our treatment of Einstein static universe in the framework of the induced matter brane gravity through the modified Friedmann equations (33) and (35). For the Einstein static universe with $H = 0$, equation (40) reduces to

$$\dot{\varpi}_\kappa = 0, \quad (41)$$

where represents that initial vector perturbations remain frozen and consequently we have neutral stability against vector perturbations.

Tensor perturbations, namely gravitational-wave perturbations, of a perfect fluid is described by the co-moving dimensionless transverse-traceless shear $\Sigma_{ab} = a\sigma_{ab}$, whose modes satisfy the following equation

$$\ddot{\Sigma}_\kappa + 3H\dot{\Sigma}_\kappa + \left[\frac{\mathcal{K}^2}{a^2} + \frac{2k}{a^2} - \frac{8\pi}{3}(1 + 3\omega)\rho \right] \Sigma_\kappa = 0, \quad (42)$$

where \mathcal{K} is the co-moving index ($D^2 \rightarrow -\mathcal{K}^2/a^2$ in which D^2 is the covariant spatial Laplacian)[33]. For the Einstein static universe, this equation by using equations (30), (32) (34) and (36), reduces to

$$\ddot{\Sigma}_\kappa + \frac{1}{a_0^2} [\mathcal{K}^2 + 2k] \Sigma_\kappa = 0. \quad (43)$$

Then, in order to have stable modes against the tensor perturbations, the following inequality should be satisfied

$$\mathcal{K}^2 + 2k > 0. \quad (44)$$

With respect to the result obtained in equation (34), representing the positivity of the spatial curvature of the Einstein static universe in induced matter brane gravity, this condition is clearly satisfied. Then, in the framework of the induced matter brane gravity, the Einstein static universe will be stable against both of the vector and tensor perturbations.

4 Conclusion

Stability issue of the Einstein static universe against scalar, vector and tensor perturbations in the context of induced matter brane gravity is studied. It is shown that in this model, the Einstein static universe has a positive spatial curvature. The stability condition against the scalar perturbations for the Einstein static universe appears as $\delta\omega_g(t) = -C\delta a(t)$ representing the variation of the time dependent geometrical equation of state parameter in terms of the variation of the cosmic scale factor. There is neutral stability against the vector perturbations, and the stability against the tensor perturbations is guaranteed because of the positivity of the spatial curvature of the Einstein static universe in induced matter brane gravity.

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